

Check-Up

17

Very large and very small numbers

- a) A teacher's personal computer has 8 Gb (gigabytes) of memory available. Which of the following would be alternative ways of expressing this:
- A eight thousand million bytes B 8 000 000 000 bytes
C 8×10^9 bytes D $8E9$
- b) A total of 8 pupils gained no marks in a sample of 2207 of those who took the Key Stage 2 handwriting test. To express this proportion as a decimal, I entered $8/2207$ onto my on-screen calculator. The answer displayed was $3.6248301E-3$. What does this mean? What percentage of the sample gained no marks?

Answers to check-up 17

- a) A, B, C and D are all different ways of expressing 8 gigabytes.
- b) It means 3.6248301×10^{-3} , which equals 0.0036248301. That's about 0.4%.

Discussion and explanation of check-up 17

The prefix *giga* means 'a thousand million', what is called a billion in the USA, and nowadays in most other countries as well. Other prefixes sometimes used for large numbers include kilo (k) for a thousand, and mega (M) for a million. So, 8 gigabytes is 8 000 000 000 bytes. Notice that I have used the convention of separating the digits into groups of 3 by using spaces. Sometimes you will see commas used for this purpose (i.e. 8,000,000,000), but this can be confusing because some countries use the comma for a decimal point.

Because it is difficult to take in the size of numbers like this at a glance, you will sometimes see them expressed in what is called *scientific notation*. To do this, we first move the decimal point so that it sits to the right of the first digit in the number. In this case, we get 8.000000000, which is actually just 8. Then you indicate with a *power* of 10, how many times you have to multiply this by 10 to get the original number. Because we have to multiply the 8 by 10 a total of nine times to get back to 8 000 000 000, we can write this big number as 8×10^9 . This is shorthand for $8 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$. Basic four-function calculators will usually just give up when the numbers get too large to display. But some calculators will use a version of scientific notation, for example, writing our number as 8E9. The E stands for 'exponent', a word that is synonymous with 'power of 10'. So, for example, we could write 123 400 000 000 as 1.234×10^{11} , but on some calculators this will be displayed as 1.234E11.

This scientific notation also works for very small numbers, but now a negative exponent is used to indicate how many times you have to *divide* by 10. So, for example, the answer to $8 \div 2207$ will be displayed on many calculators as 0.0036248. In scientific notation this is 3.6248×10^{-3} . The decimal point has been moved to the right of the first non-zero digit. This 3.6248×10^{-3} is therefore shorthand for $3.6248 \div 10 \div 10 \div 10$. Again, we note that some calculators will display this using an E to indicate the exponent, like this: 3.6248E-3. This is what my on-screen calculator does, but it is also able to give more figures after the decimal point (not that I want them).

Scientific notation is useful because the power of 10 is much more significant in indicating the size of number we are dealing with than the actual digits involved.

Summary of key ideas

- ◆ To write a large number in scientific notation, move the decimal point to the right of the leading digit and then indicate with a power of 10 how many times this has to be multiplied by 10 (e.g. $650\,000 = 6.5 \times 10 \times 10 \times 10 \times 10 \times 10 = 6.5 \times 10^5$).
- ◆ To write a very small number in scientific notation, move the decimal point to the right of the first non-zero digit and then indicate with a negative power of 10 how many times this has to be divided by 10 (e.g. $0.00065 = 6.5 \div 10 \div 10 \div 10 \div 10 = 6.5 \times 10^{-4}$).
- ◆ Some calculators use a version of scientific notation, where an E followed by a number is used to indicate the power (exponent) of ten (e.g. $6.5E5$ and $6.5E-4$).

Further practice

17.1 Arrange these approximate populations (data for 1996) in numerical order from smallest to largest:

India: 9.4×10^8 Japan: 1.3×10^8 UK: 5.8×10^7 China 1.2×10^9

17.2 The DfES proposes to provide £290 million pounds for school improvement in England. Write this amount out in full, and in scientific notation.

17.3 In the previous year to that in Check-up 17 question (b), a total of 9 pupils gained no marks in a sample of 2008 of those who took the Key Stage 2 handwriting test. To express this proportion as a decimal, I entered $9/2008$ onto my on-screen calculator. The answer displayed was $4.4820717E-3$. What does this mean? What percentage of the sample is this?